1. Use my MATLAB function on the syllabus `erwater.m` to compute the relative permittivity (also called dielectric constant) of pure water.

(a) Plot the real part of the relative permittivity, $\mathcal{R}\{\varepsilon_r\}$, versus frequency at 275 and at 300 K.

(b) How does $\mathcal{R}\{\varepsilon_r\}$ change with temperature for $f > 4 \text{ GHz}$?

(c) Plot the imaginary part of the relative permittivity, $\mathcal{I}\{\varepsilon_r\}$, versus frequency at 275 and 300 K.

(d) How does the frequency at which the imaginary part of the dielectric constant is greatest change with temperature?

(e) Plot the real and imaginary parts of the index of refraction, $n = n' - jn''$, versus frequency at 300 K.

(f) What does the real part of the refractive index represent? Think about the equation for the electric field of a propagating wave.

(g) Does the imaginary part of the refractive index peak at the same frequency as the imaginary part of the dielectric constant?

(h) Given your answer to Question 1g, at what frequency is the attenuation of microwave radiation the strongest at $T = 300 \text{ K}$?

Use a frequency range of 100 MHz to 100 GHz for Questions 1a and 1c, and a frequency range of 1 GHz to 1 THz for Question 1e. A log scale will work the best.

2. (a) Plot the drop size distribution for a light rain (5 mm per hour) and heavy rain (40 mm per hour) using the Marshall–Palmer drop size distribution function. The Marshall–Palmer distribution is described by (5.116) in UMF. Although not mentioned in the text, the $b$ parameter is related to the expected value of the drop size diameter $\langle d \rangle$ (the expected value can be thought of as the average or most common drop diameter) through the relationship $b = \frac{1}{\langle d \rangle}$.

(b) Find the total number of rain drops per cubic meter for the light and heavy rain.
(c) The Rayleigh scattering approximation can be used when \(|n_s k_o d| < 0.5\), where
\(n_s\) is the refractive index of the particles (assuming the background medium is air), \(k_o\) is the free–space wavenumber, and \(d\) is the diameter of the particle. Use the expected value of raindrop diameter, \(\langle d \rangle\), in your calculations.

i. If the temperature of the rain drops are 275 K (near freezing), for what frequencies can the Rayleigh approximation be used for the light rain?

ii. For the heavy rain?

iii. Does your answer change significantly for warmer temperatures (around 290 K)?

3. Use my function `kappa_rain.m` on the syllabus to calculate the absorption and scattering coefficients for rain, \(\kappa_a\) and \(\kappa_s\).

I derived expressions for \(\kappa_a\) and \(\kappa_s\) by assuming rain drops can be approximated as spherical particles and Rayleigh scattering is appropriate. Specifically, I used scattering cross sections given by (5.75) and (5.76) in UMF. To find the coefficients, I integrated the product of the drop size distribution function and either the absorption or scattering cross sections, \(\sigma_a\) and \(\sigma_s\), over all possible diameters

\[
\kappa = \int_0^\infty p(a) \sigma(a) \, da \tag{1}
\]

where \(\kappa\) is either the absorption or scattering coefficient, \(p(a)\) is the drop size distribution, \(\sigma(a)\) is the cross section for absorption or scattering, and \(a\) is either the radius or diameter of the particles. Note that the absorption and scattering cross sections are called \(Q_a\) and \(Q_s\) in UMF. My expression required the use of the gamma function, \(\Gamma(x)\), where

\[
\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, dt. \tag{2}
\]

Fortunately there is a MATLAB function for \(\Gamma(x)\). I checked my expressions by comparing with Figure 5.26 in UMF. Note that Figure 5.26 was created using a different drop–size distribution (the Laws–Parsons as opposed to the Marshall–Palmer) so that values obtained with my function will not be exactly the same as what you see in the figure, but they should be within a factor of 10. Why the discrepancy? Are all 25 mm rain rate (for example) rain events the same, and the same in different parts of the world? I don’t think so, and these models are definitely empirical (found to fit a specific data set). In addition, these discrepancies will depend on the wavelength.

I encourage you to derive your own expressions if you have time and the interest and check to see if I have done it correctly.

(a) What is the the extinction coefficient, \(\kappa_e = \kappa_a + \kappa_s\), and the scattering albedo, \(\omega = \kappa_s/\kappa_e\), for a rain rate of 5 mm per hour assuming a temperature of 280 K at 10 GHz?

(b) What is the the extinction coefficient and the scattering albedo for a rain rate of 25 mm per hour assuming a temperature of 280 K at 10 GHz?

(c) What is the the extinction coefficient and the scattering albedo for a rain rate of 25 mm per hour assuming a temperature of 280 K at 85 GHz?
4. Use the m–file on the course website `scat_layer.m` to explore the radiometric characteristics of a scattering layer. To properly use this function, look at Figure 1 and read the comments at the beginning of the file or type `help scat_layer` at the MATLAB command prompt. The reason for the weird format of the output is because of the numerical technique that is used to solve the equations.

Example: for height = 100 m, f = 37e9 Hz, T = 275 K, $\kappa_a = 1.7 \times 10^{-4}$ Np m$^{-1}$ and $\kappa_s = 9.6 \times 10^{-6}$ Np m$^{-1}$ (I made these up, they do not correspond to a specific rain rate of which I am aware), $T_{bo} = [0 0 0 0 0 0]$, and $z = 0$, the result $T_b = [0 0 0 18.8966 6.9841 4.9719]$ is found, which corresponds to upwelling brightness temperatures at $z = 0$ m of zero kelvin, and downwelling brightness temperatures of approximately 19 K at $103^\circ$, 7 K at $131^\circ$, and 5 K at $159^\circ$ at $z = 0$ where $0^\circ$ is defined as straight up (+z direction). For the same situation and boundary conditions, the result for $z = 100$ is $T_b = [18.8966 6.9841 4.9719 0 0 0]$, which correspond to downwelling brightness temperatures at $z = 100$ m of zero kelvin and upwelling brightness temperatures of approximately 19 K at $76^\circ$, 7 K at $49^\circ$, and 5 K at $21^\circ$. This example is illustrated in Figure 1. I find it necessary to make figures like this when interpreting the results, else I get confused. Note the symmetry in the results, and note that the scattering albedo is approximately 0.05. When the scattering albedo is this low emission and absorption (and not scattering) are the processes that primarily determine the brightness temperatures.

Answer the following questions.

(a) If all boundary conditions are 0 K and the temperature of the scattering layer is also 0 K, what are the downwelling brightness temperatures at $z = 0$ m and the upwelling brightness temperatures at $z = 100$ m if $f = 37$ GHz and the rain rate is 25 mm per hour? Explain why this makes sense.

(b) If all boundary conditions are 275 K and the temperature of the scattering layer is also 275 K, what are the downwelling brightness temperatures at $z = 0$ m and
the upwelling brightness temperatures at \( z = 100 \) m if \( f = 37 \) GHz and the rain rate is 25 mm per hour? Explain why this makes sense.

(c) If all boundary conditions are 0 K and the temperature of the scattering layer is 275 K, what are the downwelling brightness temperatures at \( z = 0 \) m and the upwelling brightness temperatures at \( z = 100 \) m if \( f = 37 \) GHz and the rain rate is 25 mm per hour? Explain the trend in brightness temperatures at both \( z = 0 \) and 100 m.

(d) Simulate rain over a prairie grassland using the following conditions: scattering layer height of 2 km; temperature of rain is 275 K; downwelling brightness temperatures of 60 K at all angles at the top of the scattering layer (atmospheric emission from above the clouds); and upwelling brightness temperatures of 290 K at all angles at the bottom of the scattering layer (approximately the brightness temperature of a grassland). For a frequency of 37 GHz and a rain rate of 25 mm per hour, report the upwelling brightness temperatures at 2 km and the downwelling brightness temperatures at the surface. Why is the change with angle at the top of this scattering layer different than for Problem 4c? (Hint: observe what happens when you try different boundary conditions). What is the scattering albedo for the scattering layer?

(e) Use the same conditions in Problem 4d but change the frequency to 10 GHz. What is the new scattering albedo? Report the brightness temperatures at 2 km and at the surface and explain why the variation with angle at 2 km is different from Problem 4d.

(f) Use the same conditions in Problem 4d but change the rain rate to 5 mm per hour. What is the new scattering albedo? Report the brightness temperatures at 2 km and at the surface and explain why the variation with angle at 2 km again the same as for Problem 4d.

(g) Make up and answer your own problem by changing boundary conditions, frequency, rain rate, height of the scattering layer, etc.

5. Read “Cellular Network Infrastructure: The Future of Fog Monitoring?” with doi:10.1175/BAMS-D-13-00292.1, focusing on the first section and “Scientific Background.” In Figure 1, think of the vertical axis as being \( \kappa_e \).

(a) What makes it difficult to use satellite instruments to detect fog?

(b) How about “visibility sensors” that use visible light, what is the main drawback?

(c) What is one particular advantage of using the commercial microwave links (MLs) used to create cellular communication networks?

(d) What change to these MLs are expected to happen in the future?

(e) Why would this change be beneficial for detecting fog?

(f) Examine Figures 5.18 and 5.21 from UMF (in the course notes). Can the Rayleigh approximation be used for fog at 30 GHz? Why or why not?