Brightness Temperature & a Homogeneous, Isothermal Halfspace

Some of the incident radiation ($T_b(z=0^+, \mu_1)$) is reflected at the boundary between medium 1 and medium 2, and some radiation is transmitted.

The amount transmitted is:

$$T_b(z=0^+, \mu_2) = T_{p1 \rightarrow 2} T_b(z=0^+, \mu_1)$$

where $\mu_1$ and $\mu_2$ are related to each other via Snell's law.

Recall that:

$$\epsilon = \frac{\text{emitted radiation}}{\text{blackbody radiation}}$$

In this case:

$$\epsilon = \frac{T_b(z=0^+, \mu_2)}{T_0} = \frac{T_{p1 \rightarrow 2} T_b(z=0^+, \mu_1)}{T_0} = \frac{T_{p1 \rightarrow 2} T_0}{T_0} = T_{p1 \rightarrow 2}$$

So:

$$T_b \text{ brightness temperature of a homogeneous and isothermal halfspace} = \epsilon T_0$$

where:

$$\epsilon_p = T_{p1 \rightarrow 2} = f(n_1, n_2) = f(\text{soil moisture}) = f(\text{ocean salinity})$$

understood to depend on polarization.
Kirchhoff's Law

What is the emissivity of a surface in general?

Thought Experiment: Imagine a surface (not necessarily flat) surrounded by a blackbody cavity at temperature $T_0$. The surface is uniform in composition and depth (homogeneous) and isothermal, temperature $T_0$.

Kirchoff's Law - not necessarily quasi-specular

$T_p^+(k_r) = \text{emission from surface} + \text{radiation scattered by surface into } k_s$

\[ T_p^+(k_r) = \frac{e_p(k_s) - T_0}{R_{pq}(k_r, k_i)T_0} \]

Scattered radiation

\[ dT_p^+(k_r) = \text{one "piece" of scattered radiation} \]

\[ = R_{pq}(k_r, k_i)T_p^-(k_i) \]

\[ = p-polarization radiation scattered into } k_r \text{ from q-polarization incident radiation traveling in } k_i \]

\[ T_p^+(k_r) = \frac{\text{total scattered radiation}}{2} = \frac{\sum_{q=h, v} \int R_{pq}(k_r, k_i)T_0(k_i) d\Omega_i}{2} \]

- note that $R_{pq} \neq 0$ for $k_r = -k_i$

since surface is not quasi-specular!

When $k_r = k_s$

\[ T_p^+(k_s) = e_p(k_s)T_0 + \sum_{q=h, v} \int R_{pq}(k_s, k_i)T_0(k_i) d\Omega_i = \{e_p(k_s) + \sum_{q=h, v} \int R_{pq}(k_s, k_i) d\Omega_i\}T_0(k_s) \]

Because this is a blackbody cavity

\[ T_p^+(k_s) = T_p^-(k_i) = T_0 \]

(else temperature of blackbody would be changing)

Then, $T_0 = \{e_p(k_s) + \sum_{q=h, v} \int R_{pq}(k_s, k_i) d\Omega_i\}^2 T_0$

Kirchhoff's law

\[ e_p(k_s) = 1 - \sum_{q=h, v} \int R_{pq}(k_s, k_i) d\Omega_i \]
Kirchhoff's law: $e_p(k_o) = 1 - \sum_{q>1} \sum_{m} R_p(q, k_i) \delta \psi_i$

The emissivity at polarization $p$ = 1 - the sum of the reflectivities in all possible directions that transfer incident $p$-pol or $q$-pol radiation into $p$-pol radiation in the $k_o$ direction.

For a quasi-specular surface:

$\sum_{q=1}^{\infty} \sum_{m} R_p(q, k_i) \delta \psi_i = R_{p_{1\rightarrow 2}}(k_o - k_i)$

and then:

$e_p(k_o) = e_p(\theta) = 1 - R_{p_{1\rightarrow 2}}(\theta)$

Since all reflectivities except for the specular direction ($k_i = -k_o$) are zero!

$m$ makes sense since:

$e_p(\theta_i) = T_{p_{1\rightarrow 2}}(\theta_i)$

$= 1 - R_{p_{2\rightarrow 1}}(\theta_i)$

And since it turns out that $R_{p_{2\rightarrow 1}}(\theta_i) = R_{p_{1\rightarrow 2}}(\theta_i)$, we don't prove it, but check it out!

$e_p(\theta_i) = e_p(\theta) = 1 - R_{p_{1\rightarrow 2}}(\theta)$

Example: What is the emissivity of liquid water?

$e_p(\theta) = ?$

$e_p(\theta) = 1 - R_{p_{1\rightarrow 2}}(\theta) = 1 - |\Gamma_p(\theta)|^2$

For h-pol:

$|\Gamma_h|^2 = \left| \frac{n_1 \cos \theta - n_2 \cos \left[ \arcsin \left( \frac{n_1 \sin \theta}{n_2} \right) \right]}{n_1 \cos \theta + n_2 \cos \left[ \arcsin \left( \frac{n_1 \sin \theta}{n_2} \right) \right]} \right|^2$

And what is the h-pol brightness temperature if $T_o = 300$ K?

<table>
<thead>
<tr>
<th>$\Theta$</th>
<th>$e_h$</th>
<th>$T_b = e_h T_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.36</td>
<td>108 K</td>
</tr>
<tr>
<td>15°</td>
<td>0.35</td>
<td>105 K</td>
</tr>
<tr>
<td>30°</td>
<td>0.32</td>
<td>96 K</td>
</tr>
<tr>
<td>60°</td>
<td>0.20</td>
<td>60 K</td>
</tr>
</tbody>
</table>

Write that:

$\text{since } T_b < T_o$,

$\text{water + nair}$

(there is a boundary)
Rough Surfaces

For specular (perfectly flat) and quasi-specular (effectively flat) surfaces, all of the energy is scattered in the specular direction, \( \Theta \).

When the surface is rough, energy is scattered in several directions.

What is a surface "rough"? It depends on the electrical size and the roughness.

If \( A \gg \sigma_{\text{surface}} \), where \( \sigma_{\text{surface}} \) = standard deviation of surface height, then the surface may be considered to be quasi-specular.

**Kirchoff's law**

\[
\varepsilon_p(\Theta) = 1 - \sum_{q=h,v} R_p(q, \Theta') d\Omega'
\]

Again, \( R_p(q, \Theta') = 0 \) for \( \Theta \neq \Theta' \)

**Empirical model**

\[
\varepsilon_p^{\text{rough}}(\Theta) = 1 - R_p^{\text{rough}}(\Theta), \quad R_p^{\text{rough}}(\Theta) = R_p(\Theta) e^{-h}
\]

where \( h \) = "roughness parameter", \( 0 \leq h \leq 0.6 \) (approximately)

**Note:** \( \varepsilon_p^{\text{rough}}(\Theta) > \varepsilon_p(\Theta) \) since \( R_p^{\text{rough}}(\Theta) < R_p(\Theta) \)

And note:

\[
\sum_{q=h,v} \int R_p(q, \Theta') d\Omega' = R_p^{\text{rough}}(\Theta) < R_p(\Theta) \quad \text{specular reflectivity}
\]

This is because some of the scattered radiation gets absorbed.

In the scientific literature, \( h \) has been related to \( \sigma_{\text{surface}} \), \( \Theta \), soil moisture (for soil surface), ...

"perfectly rough surface": \( R_p^{\text{rough}}(\Theta) = 0 \), \( \varepsilon_p^{\text{rough}} = 1 \)