Case Studies for Microwave Radiometry

Remote Sensing & the Atmosphere
1) total precipitable water
2) water vapor profile, temperature profile
3) precipitation

Terrestrial (Earth surface) Remote Sensing
4) sea surface salinity
5) soil moisture and vegetation water content

Recall: Attenuation of Electromagnetic Waves

Recall a spherical wave propagating away from a source.

We postulated that we can use a plane wave with $E_0$ and $E_f$ components to approximate a real spherical wave, for a small view angle at a specific $(\theta, \phi)$. 
In general, \( \frac{e^{-jkR}}{R} = \frac{e^{-j(k_0-k')R}}{R} = e^{-jkr} e^{-jkR} \). 

spherical propagation factor for energy conservation.

\( S_{\text{avg}} = \text{propagating power} = \frac{1}{2} \text{Re}\{E \times E^*\} \)

\( = \frac{|E|^2}{2} = \frac{|E_0|^2 + |E|^2}{2} = \frac{R}{2} \frac{1}{k^2} \frac{1}{R^2} e^{-2k''R} \)

\( \text{note: } 2k'' \)

\( |S_{\text{avg}}| = R S_{R} (R) e^{-kR} \) where \( kR = 2k'' = 2\kappa'' \) is the attenuation coefficient.

\( \text{Greek letter kappa, \( \kappa \) is for "attenuation".} \)

Recall that \( R^2 S_R = \text{radiation intensity, } \text{W}\cdot\text{sr}^{-1} \), since \( [S_R] = \text{W}\cdot\text{m}^{-2} \).

Therefore \( R^2 S_R e^{-kR} \) describes the attenuation of radiation intensity.

Now consider radiation intensity per radiating area (and perhaps per wavelength or per frequency); we call these quantities the brightness and spectral brightness.

So...

\( B_\lambda (R) = B_\lambda (R=0) e^{-kR} \)  \( \text{(1)} \)

where \( R=0 \) is some arbitrary reference point.

Note that

\( \frac{B_\lambda (R)}{B_\lambda (R=0)} = e^{-kR} \)

\( \ln \left\{ \frac{B_\lambda (R)}{B_\lambda (R=0)} \right\} = \ln (e^{-kR}) = -kR \)

\( \int_{u=0}^{u=R} \frac{dB_\lambda}{B_\lambda} = \int_{u=0}^{u=R} -kR \text{d}u \)

\( B_\lambda = B_\lambda (R=0) \)

When there is an exponential decrease (1), this means that attenuation is directly proportional to the current value and the quantity!

So...

\( \frac{dB_\lambda (r)}{B_\lambda (r)} = -kR \text{d}r \text{ and } dB_\lambda (r) = -kR B_\lambda (r) \text{d}r \)  \( \text{(2)} \)
Optical Depth

\[ \frac{dB_r(r, \hat{r})}{dr} = -K_r B_r(r, \hat{r}) \]  

This describes the attenuation of spectral brightness.

\[ dB_r(r, \hat{r}) = -K_r B_r(r, \hat{r}) \, d\hat{r} \]  

Now, integrate from \( r' \) to \( r \) for \( \hat{r} \):

\[ \int_{r'}^{r} \frac{dB_r(u, \hat{r})}{B_r(u, \hat{r})} \, du = \int_{r'}^{r} -K_r \, du = -T(r', r) \]  

Where:

\[ T(r', r) = \text{the optical depth from } r' \text{ to } r \equiv \int_{r'}^{r} K_r(u) \, du \]  

Using (4):

\[ \int_{r'}^{r} \frac{dB_r(u, \hat{r})}{B_r(u, \hat{r})} = \ln \left( \frac{B_r(r, \hat{r})}{B_r(r', \hat{r})} \right) = \ln \left( \frac{B_r(r, \hat{r})}{B_r(r', \hat{r})} \right) = -T(r', r) \]

and after taking the exponential of each side:

\[ B_r(r, \hat{r}) = \text{spectral brightness at point } r \text{ in the } \hat{r} \text{ direction} \]

\[ = B_r(r', \hat{r}) e^{-T(r', r)} \]

The optical depth is interpreted as the amount of attenuation that occurs over the distance from point \( r' \) to point \( r \).

\[ T(r', r) \] is the integrated attenuation from \( r' \) to \( r \).

- Large \( T \) → lots of attenuation
- Small \( T \) → little attenuation
Radiative Transfer

In a non-scattering medium (e.g., a clear atmosphere at microwaves frequencies) when \( \lambda \gg d \), and \( d \) is the characteristic dimension (representative size) of scatterers (inhomogeneities) in the medium, the change in spectral brightness depends on four things:

1) the current level of spectral brightness
2) the properties of the medium
3) the path length or distance through which the spectral brightness travels
4) new radiation in the form of emission from the medium itself

\[
\frac{dB_x(r,\mathbf{s})}{dr} = \text{change in spectral brightness of } r \text{ in the } \mathbf{s} \text{ direction} = -k_x B_x(r,\mathbf{s}) \, dr + J(r,\mathbf{s}) \, dr \quad (5)
\]

What is \( J(r,\mathbf{s}) \)?

Consider a medium emitting at temperature \( T \). \( dr \) must emit according to its temp

\[
\text{illuminate } dr \text{ with blackbody spectral brightness from adjacent } dr
\]

When in local thermodynamic equilibrium, power into \( dr \) = power out of \( dr \)

\[
\frac{dB_x(r,\mathbf{s})}{dr} = 0 = -k_x B_x^b(r,\mathbf{s}, T) \, dr + J(r,\mathbf{s}) \, dr \rightarrow J(r,\mathbf{s}) = k_x B_x^b(r, T)
\]

What is absorbed must be emitted! Consider \( k_x = 0 \) for all \( \mathbf{s} \) except \( s_1 \). Whatever is absorbed at \( s_1 \), the same amount must be emitted at \( s_1 \)!

In general,

\[
\frac{dB_x(r,\mathbf{s})}{dr} = -k_x B_x(r,\mathbf{s}) \, dr + k_x B_x^b(r, T) \, dr
\]

Now substitute \( B_x^b(r,\mathbf{s}) = \frac{2k_x}{\pi^2} T_x^2(r,\mathbf{s}) = \frac{2k_x}{\pi^2} T_x(r,\mathbf{s}) \) into (6).

\[
\left\{ \frac{2k_x}{\pi^2} T_x(r,\mathbf{s}) \right\}' = -k_x \left\{ \frac{2k_x}{\pi^2} T_x(r,\mathbf{s}) \right\} dr + k_x \left\{ \frac{2k_x}{\pi^2} T_x(r) \right\} \, dr
\]

\[
\left( \frac{dT_x(r,\mathbf{s})}{dr} \right)' = -k_x T_x(r,\mathbf{s}) \, dr + k_x T_x(r) \, dr
\]
General Solution of Radiative Transfer

\[ \frac{d T_e(r, \lambda)}{d r} = -k \alpha T_e(r, \lambda) + kT(r) \]

\( k \alpha = \) attenuation via absorption, \( N \) P\cdot m\(^{-1}\)

\( T(r) = \) thermal temperature at \( r \), K

Substitute \( T = \int k \alpha \, dr \), \( dT = k \alpha \, dr \), \( dr = \frac{dT}{k \alpha} \) in (7):

\[ \frac{d T_e(r, \lambda)}{dT} + k \alpha T_e(r, \lambda) = k \alpha T(r) \]

Multiply both sides of (8) by \( e^{T(0, r)} \), an "integration factor":

\[ \frac{dT_e(r, \lambda)}{dT} e^{T(0, r)} + T_e(r, \lambda) e^{T(0, r)} = T(r) e^{T(0, r)} \]

\[ \frac{dT_e(r, \lambda)}{dT} \int_0^r T_e(r', \lambda) e^{T(0, r')} \, dr' = T(r) e^{T(0, r)} - T_e(0, \lambda) \]

Now integrate (9) from 0 to \( r \):

\[ T_e(r, \lambda) = T_e(0, \lambda) e^{-T(0, r)} + \int_0^r k \alpha(r') T(r') e^{-T(r', r)} \, dr' \]

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General solution of radiative transfer for non-scattering media.

Graphically:

- Original spectral brightness gets attenuated from \( 0 \) to \( r \).
- Emission of spectral brightness at each \( r \) also gets attenuated as it travels from \( r \) to \( r' \).