Microwave Radiometry

Radiometry is the measurement of electromagnetic radiation. Usually this is just the power associated with the radiation. All materials (gases, liquids, solids) radiate (or "emit") via the process of thermal emission. Radiometry can be used to infer the properties of a material in the following way. If the temperature of a material is known (and thus the "potential emission"), by understanding how the material alters/modifies thermal emission through scattering and/or attenuation we can solve the inverse problem (see Bohnen and Huddman figure). If the scattering and/or attenuation is already known precisely, then radiometry can tell us about the temperature of the material.

Microwave radiometry: a remote sensing technique that employs the measurement of thermally-generated radiation in the microwave region of the electromagnetic spectrum.

Radiometric Quantities

- **Radiant Flux or Power (W)**: radiant energy transmitted, emitted, scattered, or received per time.
- **Radiant Flux Density or Power Density (W/m²)**: radiant flux per area, also called radiant emittance (radiant flux density emitted by a surface) or irradiance. (radiant flux density incident on a surface).
- **Radiant Intensity or Radiation Intensity (W/sr)**: radiant power per solid angle.
- **Radiance or Brightness (W/m²·sr⁻¹)**: radiant power per area per solid angle.
- **Spectral Radiance or Spectral Brightness**:
  - \[ B_\lambda (\lambda) = \text{W/m}^2 \cdot \text{sr}^{-1} \cdot \text{m}^{-1} \]
  - \[ B_\xi (\xi) = \text{W/m}^2 \cdot \text{sr}^{-1} \cdot \text{Hz}^{-1} \]

\[ B(\theta, \phi) = \frac{W}{W^2 \cdot \text{sr}} \]
What is the source of brightness?

Because all matter above absolute zero (0 K) contains accelerating charges, all matter emits radiation!

Recall the wave/particle duality. Instead of electromagnetic waves, we can also consider electromagnetic radiation in the form of photons. Photons are emitted or absorbed as a consequence of discrete energy transitions.

1. **Electron Transitions**
   - Electron "jumps" to a higher energy level (excited state).
   - Energy absorbed or emitted corresponds exactly to difference in energy between energy levels.
   - A photon is emitted through spontaneous emission.

2. **Vibrational, Rotational, Translational Transitions**
   - Energy state of molecule changes.
   - Absorbs a photon of frequency $f_1$.
   - E = photon energy = $hf$
     - $h$ = Planck constant = $6.626 	imes 10^{-34}$ J·s
   - Emits a photon of frequency $f_2$.

In the case of a material with an infinite number of transitions spaced throughout the spectrum, the material is called a perfect radiator (can radiate at all frequencies) and a perfect absorber (will absorb all incident radiation). Such a material is called a blackbody. No blackbodies exist in nature, but some materials are approximately a blackbody in certain frequency bands. A blackbody is a theoretical construct that serves as a reference.

**Blackbody: perfect radiator and perfect absorber!**
Blackbody thermal emission is described by the Planck law:

\[ B_\nu (\nu, T) = \frac{2\hbar \nu^3}{c^2} \frac{1}{e^{\frac{\hbar \nu}{kT}} - 1} = \text{spectral brightness emitted by a blackbody} \]

where:
- \([B_\nu (\nu, T)] = \text{W} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \cdot \text{Hz}^{-1}\]
- \(\hbar = \text{Planck constant} = 6.63 \times 10^{-34} \text{J} \cdot \text{s}\)
- \(\nu = \text{frequency of radiation} \cdot \text{Hz}\)
- \(c = \text{speed of light in vacuum} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \text{m} \cdot \text{s}^{-1}\)
- \(k = \text{Boltzmann constant} = 1.38 \times 10^{-23} \text{J} \cdot \text{K}^{-1}\)
- \(T = \text{temperature of blackbody} \cdot \text{K}\)

Note that blackbody spectral brightness is a function of temperature!

Specifically, \(B_\nu (\nu, T)\) increases at all frequencies when temperature increases!

The Planck law in terms of wavelength:

\[
\int_0^\infty B_\nu (\nu, T) \nu \, d\nu = \frac{1}{\pi^2} \int_0^\infty B_\lambda (\lambda, T) \lambda \, d\lambda = \frac{\hbar}{c^3} \int_0^\infty B_\lambda (\lambda, T) \lambda \, d\lambda = \frac{\hbar}{c^3} \int_0^\infty B_\lambda (\lambda, T) d\lambda
\]

To relate \(B_\nu (\nu, T)\) to \(B_\lambda (\lambda, T)\) we must relate \(d\nu\) to \(d\lambda\).

\[ c = \lambda \nu, \quad f = \frac{c}{\lambda} = c \lambda^{-1}, \quad df = -c \lambda^{-2} d\lambda = -\frac{c}{\lambda^2} d\lambda \]

\[
\int_0^\infty B_\nu (\nu, T) \, d\nu = \int_0^\infty B_\lambda (\lambda, T) \left[ -\frac{c}{\lambda^2} d\lambda \right] = \int_0^\infty B_\lambda (\lambda, T) d\lambda = \int_0^\infty B_\lambda (\lambda, T) d\lambda
\]

\[ B_\nu (\nu, T) \frac{c}{\lambda^2} = B_\lambda (\lambda, T) \]

\[ B_\lambda (\lambda, T) = \frac{2\hbar c^2}{\lambda^5} \frac{1}{e^{\frac{\hbar c}{\lambda kT}} - 1} \]

This form is often used in atmospheric science. Why do you think this is so?
What is the frequency \( f \) maximum spectral brightness?

\[
\frac{dB_\lambda}{dT} = 0 \quad \Rightarrow \quad f_{\text{max}} = 5.87 \times 10^8 T \\
\text{where} \quad [f_{\text{max}}] = \text{Hz}, \quad [T] = \text{K}
\]

What is the wavelength \( \lambda \) maximum spectral radiance?

\[
\frac{dA_\lambda}{dT} = 0 \quad \Rightarrow \quad \lambda_{\text{max}} T = 2.897 \times 10^{-3} \\
\text{where} \quad [\lambda_{\text{max}}] = \text{m}, \quad [T] = \text{K}
\]

Note that \( \lambda_{\text{max}} \neq \frac{c}{f_{\text{max}}} \). Why? I don't have a physical explanation!

Planck law approximations

When \( \frac{hf}{kT} \gg 1 \), then \( \frac{1}{e^{\frac{hf}{kT}} - 1} = \frac{1}{e^{\frac{hf}{kT}}} = e^{-\frac{hf}{kT}} \)

and consequently

\[
B_\lambda (f, T) = \frac{2hf^3}{c^2} e^{-\frac{hf}{kT}} \quad \text{(Planck law)}
\]

Valid for high frequencies at typical Earth temperatures

When \( \frac{hf}{kT} \ll 1 \) then \( e^{\frac{hf}{kT}} - 1 \approx \frac{hf}{kT} \) (you'll show this in PS5)

and consequently

\[
B_\lambda (f, T) = \frac{2kT}{c^2} \quad \text{(Rayleigh-Jeans law)}
\]

Valid for low frequencies at typical Earth temperatures

Normally microwave remote sensing scientists will write the Rayleigh-Jeans law

\[
B_\lambda (f, T) = \frac{2kT}{\lambda^2} \quad \text{since} \quad \frac{f^2}{c^2} = \frac{1}{\lambda^2}
\]

However, the Rayleigh-Jeans law as a function of wavelength (not frequency) is

\[
B_\lambda (\lambda, T) = \frac{2ckT}{\lambda^4}
\]

Recall: \( d\lambda = \frac{1}{\lambda^2} d\lambda \)

\[
B_\lambda (\lambda, T) = 2\frac{c}{\lambda^4} T \quad \Rightarrow \quad d\lambda = \frac{1}{\lambda^2} d\lambda
\]
The diagram illustrates the relationship between brightness and frequency for various temperatures. The x-axis represents frequency in Hz, while the y-axis represents brightness in W m^-2 Hz^-1 sr^-1. Different temperature lines are shown, ranging from 100 K to 10 Billion K, with temperature increments of 10 Million K. The diagram also indicates different regions of the electromagnetic spectrum, such as radio, infrared, optical, X-ray, and ultraviolet.
#1 on Problem Set 5

what it should look like
6. Thermal Emission

Fig. 6.2: Blackbody emission curves at temperatures typical of the sun and of the earth and atmosphere. (a) The actual value of Planck's function, plotted on a logarithmic vertical axis. The diagonal dashed line corresponds to Wien's law (6.3). (b) Normalized depictions of the same functions as in (a), so that the areas under each curve are equal. Note that the vertical axis in this case is linear.
Figure 2-12. Spectral radiant emittance of a blackbody at various temperatures. Note the change of scale between the two graphs.

Section 4.1 The Origin and Nature of Radiation

Figure 4.1 Spectral distribution of radiant energy from a full radiator at a temperature of (a) 6000 K, left-hand vertical and lower horizontal axis, and (b) 300 K, right-hand vertical and upper horizontal axis. About 10% of the energy is emitted at wavelengths longer than those shown in the diagram. If this tail were included, the total area under the curve would be proportional to $\sigma T^4$ (W m$^{-2}$). $\lambda_m$ is the wavelength at which the energy per unit wavelength is maximal.
Absorption and Scattering of Light by Small Particles

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